# Scheduling in Wireless Networks with Rayleigh-Fading Interference

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**Abstract**—We study approximation algorithms for optimization of wireless spectrum access with $n$ communication requests when interference conditions are given by the Rayleigh-fading model. This model extends the deterministic interference model based on the signal-to-interference-plus-noise ratio (SINR) using stochastic propagation to address fading effects observed in reality. We consider worst-case approximation guarantees for the two standard problems of capacity maximization and latency minimization. Our main result is a generic reduction of Rayleigh fading to the deterministic non-fading model. It allows to apply existing algorithms for the non-fading model in the Rayleigh-fading scenario while losing only a factor of $O(\log^* n)$ in the approximation guarantee. This way, we obtain the first approximation guarantees for Rayleigh fading and, more fundamentally, show that non-trivial stochastic fading effects can be successfully handled using existing and future techniques for the non-fading model. We generalize these results in two ways. First, the same results apply for capacity maximization with variable data rates, when links obtain (non-binary) utility depending on the achieved SINR. Second, for binary utilities, we use a more detailed argument to obtain similar results even for distributed and game-theoretic approaches. Our analytical treatment is supported by simulations illustrating the performance of regret learning and, more generally, the relationship between both models.

**Index Terms**—wireless network, transmission scheduling, SINR, Rayleigh fading.

## 1 Introduction

In wireless networks, throughput can be significantly increased when using efficient algorithms and protocols to coordinate and schedule transmissions. In many cases, however, the problem of scheduling transmissions to maximize throughput or minimize delay poses optimization problems that are computationally hard to solve. In this case, we can only hope to obtain approximate solutions in reasonable time.

A standard criterion in algorithmic research to benchmark optimization algorithms computing approximate solutions is the notion of *approximation factor*: For each given problem instance, the value of the computed solution is compared to the value of the optimal solution. The approximation factor of an algorithm is then taken to be the maximum factor obtained on any problem instance. For optimizing wireless interference, we often have additional information about structural properties in realistic instances that can be used to improve this worst-case perspective, such as, structure of transmission powers, distances between communication requests, or other structural properties related to, e.g., underlying metric spaces. In these cases, it is often possible to give more informative bounds on approximation factors that are parameterized in terms of structural properties.

Most algorithmic research dealing with wireless interference uses rather simple interference models, which are often based on graphs. Since the seminal work of Moscibroda and Wattenhofer [2], however, attention turned to more realistic models based on constraints involving the signal-to-interference-plus-noise ratio (SINR). This resulted in a variety of insights into the algorithmic challenges and limitations [3], [4]. Particularly, it turned out that significantly different techniques than in graph-based models have to be applied to get any non-trivial approximation factors.

While these SINR models represent a significant improvement over previous approaches, they still use a limited view of signal propagation. The main assumption is that any signal transmitted at power level $p$ is always received after distance $d$ with strength $p/d^\alpha$, for some $\alpha > 0$. In contrast, in reality signal propagation is by no means deterministic. For instance, the SINR model does not account for short-term fluctuations such as fading. There exist advanced models using stochastic approaches that take fading effects into account. Most prominently, in the Rayleigh-fading model, signal strength is modeled by an exponentially distributed random variable with mean $p/d^\alpha$. Stochastic propagation represents a major technical complication in the definition of interference models, and this may be a reason that – up to our knowledge – there are no general algorithmic results for request scheduling in this model or even for a direct comparison between the non-fading and Rayleigh-

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fading model.

In this paper, we examine the relationship between the non-fading SINR model and the Rayleigh-fading model. Our first main result is a fundamental relation between the models for instances of the same topology. It is based on a detailed analysis of the success probability in the Rayleigh-fading model, and it turns out to allow a surprisingly simple handling of the complicated stochastic propagation. This allows us to transfer existing algorithms and their performance bounds in the SINR model to the Rayleigh-fading model.

Our second main result uses a more detailed reduction to show that a similar result applies even for distributed capacity maximization via distributed regret-learning techniques. As the considered sequences generalize Nash equilibria, this result transfers the respective game-theoretic studies [5]. Our analytical results are supported by simulations illustrating the performance of regret learning and, more generally, the relationship between both models.

On a more fundamental level, our results highlight the inherent robustness of the techniques and bounds derived for the non-fading SINR model. The rather direct adaptation of existing algorithms to Rayleigh fading raises the hope that algorithms and their analyses can also be applied accordingly to interference models capturing further realistic properties.

### 1.1 Our Contribution

For a set of $n$ communication requests in the non-fading model, we consider the two prominent problems of *capacity maximization* (maximizing the number of simultaneously successful requests in a single slot) and *latency minimization* (minimizing the number of slots such that every request has been successful at least once). In the Rayleigh-fading model, interference becomes stochastic, and thus capacity maximization becomes maximizing the expected number of successful requests in a single slot. Similarly, in latency minimization we strive to minimize the expected number of slots until every request has been successful at least once. In this sense, we adapt a similar perspective as in worst-case analysis of randomized algorithms – we strive to bound the expected performance of the algorithms in an arbitrary (worst-case) topology.

In fact, for our analysis we consider a more advanced approach to capacity maximization that goes beyond a binary valuation for transmission. Instead, each request is assumed to obtain a data rate depending on the transmission quality or, more specifically, its obtained SINR. We capture this scenario using a suitable utility function for each request that depends on the obtained SINR. Then, the capacity is given by the sum of utilities of the requests. In this way, we can address the optimization of Shannon capacity in the network, among many other things. Thus, our results clearly stretch beyond the standard notion of capacity maximization considering a fixed SINR threshold.

Our first main result characterizes the probability of a request to reach a certain SINR in the Rayleigh model. This probability is never 0, and thus requests can still be successful if in the non-fading model this is completely impossible (e.g., due to extremely large noise). For a meaningful comparison in terms of approximation factors, we thus focus on interference-dominated scenarios with reasonable noise conditions (for a formal definition see below). Under these conditions, we show in Section 3 that for every set of successful requests reaching a certain SINR with respect to the non-fading model, in the Rayleigh model in expectation a constant fraction of these requests remains successful. Hence, we can use algorithms for capacity optimization in the non-fading model and lose only a constant factor when translating the output to Rayleigh fading. To bound approximation factors, however, we have to relate this to the Rayleigh-fading optimum, i.e., the maximum expected number of successful requests for any subset of transmitting requests. Here we show in Section 5 that this expected number can only be a factor of $O(\log^* n)$ larger[1] than the maximum number of successful requests in the non-fading model. This allows to use existing algorithms and their bounds to derive approximation factors in the Rayleigh-fading model. For capacity maximization, we show, e.g., an $O(\log^* n)$-approximation with power control based on [6] and with distance-based power assignments based on [7]. For latency minimization similar arguments can be applied for algorithms that use repeated single-slot success maximization [8] or ALOHA-style protocols [9] in the non-fading model. For instance, we obtain an $O(\log^* n \cdot \log n)$-approximation for uniform power assignments based on [8]. The algorithms for latency minimization allow to directly apply multi-hop scheduling techniques as in [6], [9], [10]. The transformation does not modify transmission powers or depend on metrical properties of the distances. Thus, the respective properties of the algorithms and also the lower bounds, e.g., on power control [3], [4] are preserved.

In Section 6 we consider distributed approaches for capacity maximization, namely regret-learning algorithms [11]. Here we are not able to plug in the results for the non-fading model in a similar black-box fashion. Instead, we have to argue in a more detailed way to show that for uniform power assignments the expected number of successful requests is only a constant factor smaller than the size of the non-fading optimum. The bound is again completed using previous arguments, and we obtain a $O(\log^* n)$-factor with respect to the Rayleigh-fading optimum. Note that $\log^* n$ is essentially "almost constant". However, deriving a (provable) constant bound remains open.

Finally, we conduct a number of experiments that highlight the relation between the two models and the

1. Recall that $\log^*$ denotes the iterated-logarithm function. For an introduction to the $O/\Theta/\Omega$ notation capturing asymptotic behavior of functions, we refer to, e.g., any standard textbook on design and analysis of algorithms.

performance of regret learning. In particular, we observe that with probabilistic spectrum access, the curve for success probability in the Rayleigh-fading model can be seen as a smoothed variant of the success curve in the non-fading model. We observe that the non-fading model predicts more success if total interference is small, while Rayleigh fading allows more requests to become successful if interference is large. Regret-learning algorithms show fast convergence and good performance in both models, and the number of successful requests predicted by the non-fading model is somewhat larger.

### 1.2 Related Work

In a seminal paper, Gupta and Kumar [12] study the capacity of a wireless peer-to-peer network with a random topology based on non-fading SINR constraints. This brought about a lot of further work in randomly distributed networks [13]–[15]. Similar studies have been carried out for the case of regular topologies [16], [17]. Partly, this kind of research has also been generalized to networks with fading channels. For example, Liu and Haenggi [18] consider the capacity of square, triangle, hexagon, and random networks under Rayleigh-fading interference. More often Rayleigh fading is only used to model effects of noise, and interference inside the network itself is neglected [19], [20]. This represents an orthogonal approach because we concentrate on the particular problem of coordinating simultaneous transmissions. To the best of our knowledge, a direct comparison between the non-fading and Rayleigh-fading model like it is done in this paper has not been discussed in literature yet.

Real-world networks are typically neither random nor regular. This motivates the study of arbitrary topologies, as first done by Moscibroda and Wattenhofer [2]. Following this work, approximation algorithms in the non-fading SINR world were treated quite intensively, especially for the pure scheduling problems. Important milestones for capacity maximization are constant-factor approximations for uniform transmission powers [8]. A more sophisticated approach is selecting powers based on the distance between the sender and the respective receiver [7]. For uniform power assignments a distributed algorithm has also been developed [11] that uses regret learning. For latency minimization a distributed, ALOHA-like protocol has been analyzed [9], [21]. It yields an approximation factor of $O(\log n)$ with high probability.

The probably most natural extensions are the combined problems of scheduling and power control. This is, power levels are not fixed but have to be selected by the algorithm. This offers an additional freedom to the optimal solution as well. Using uniform transmission powers yields an $O(\log \Delta)$-approximation factor [5]. Here, $\Delta$ denotes the ratio of the maximal and the minimal distance between a sender and the respective receiver. One gets $O(\log \log \Delta + \log n)$-approximations when using square-root power assignments [4], i.e. a link of length $d$ is assigned a transmission power proportional to $\sqrt{d^\alpha}$. The given approximation factors have been shown to be asymptotically almost optimal when restricting to these power assignments. However, for non-oblivious power assignments, where the power does not depend on the length of the link, even a constant-factor approximation exists [6].

## 2 Formal Model Description

We assume that our network consists of $n$ communication links $(s_1, r_1), \ldots, (s_n, r_n)$, each consisting of a sender and a receiver. In general, we do not make any assumptions on the geometry or distribution of the network nodes.

For the propagation, we consider Rayleigh-fading channels. That is, if a signal is transmitted by sender $s_j$, it is received by receiver $r_i$ at a strength of $S_{j,i}$. $S_{j,i}$ is an exponentially distributed random variable with mean $\bar{S}_{j,i}$. As usual, we assume this stochastic process to be independent for different $(j,i)$ and different time slots.

The transmission between sender $s_i$ and receiver $r_i$ achieves a data rate depending on the SINR $\gamma_i^R$, which is given by

$$\gamma_i^R = \frac{S_{i,i}}{\sum_{j \neq i} S_{j,i} + \nu} \quad .$$

Here, $\nu \geq 0$ is a constant denoting ambient noise.

Each link obtains a utility from achieving a data rate. In particular, as the data rate is proportional to the SINR $\gamma_i^R$, we assume utility is given by a function $u_i\left(\gamma_i^R\right) \geq 0$. The objective of the capacity-maximization problem is to maximize the expected sum of utilities $\mathbf{E}\left[\sum_i u_i\left(\gamma_i^R\right)\right]$. The simplest case of capacity maximization studied frequently in the theoretical computer science literature is recovered by assuming *binary utilities* as

$$u_i(\gamma_i^R) = \begin{cases} 1 & \text{if } \gamma_i^R \geq \beta \\ 0 & \text{otherwise} \end{cases}$$

This implies that the (only) goal of every link is to obtain any data rate corresponding an SINR above the global constant threshold $\beta$.

In the standard non-fading propagation model the received signal strength is always (deterministically) $\bar{S}_{j,i}$. To distinguish SINRs in both models we denote the SINR for the non-fading propagation model by $\gamma_i^{\text{nf}}$. More formally,

$$\gamma_i^{\text{nf}} = \frac{\bar{S}_{i,i}}{\sum_{j \neq i} \bar{S}_{j,i} + \nu} \quad .$$

The corresponding optimization problem in the non-fading model is then to maximize the (deterministic) sum of utilities $\sum_i u_i\left(\gamma_i^{\text{nf}}\right)$.

We compare the value of approximate solutions and optima for the corresponding optimization problems in Rayleigh and non-fading models. As one can easily see, this comparison might be quite unfair, even for binary utilities. Suppose a large noise value $\nu$ dominates all signal strength means $\bar{S_{i,i}}$. In this case, the transmission cannot reach *any* reasonable SINR values in the non-fading model, even in the absence of interference. In the Rayleigh-fading model in contrast, a small probability to reach any SINR remains. Thus, the value of any solution in the non-fading model is very close to or exactly 0. This implies that the Rayleigh model is infinitely better, as the relative approximation factor to the solution value in the Rayleigh model becomes arbitrarily large.

This problem occurs only with large noise. In contrast, our focus is to analyze scheduling algorithms and the impact of interference. Therefore, we focus on the more interesting case, in which the noise is not too large. Put differently, we assume that for every link $i$ the signal strength $\bar{S_{i,i}}$ is large enough to obtain an SINR against the noise that yields significant utility. Equivalently, we formulate this condition as a requirement on the utility function. $u_i$ is allowed to be small only for SINR values up to a suitable threshold, and then is non-decreasing and concave with growing SINR. Formally, we restrict our attention to valid utility functions given as follows.

**Definition 1.** *For a link $i$ in a given network, a non-negative function $u_i : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is a* valid utility function *if there exist a constant $c_i > 1$ such that $u_i$ is non-decreasing and concave on the interval $[\bar{S_{i,i}}/(c_i \nu), \infty)$.*

Observe that these utility functions encompass most of the capacity maximization problems considered in the literature. To highlight this property we consider three prominent examples.

- The binary utilities outlined above constitute valid utility functions for a tuple $(c, \beta)$, where $c > 1$ is some constant and $\beta \leq \frac{\min_i \bar{S_{i,i}}}{c\nu}$. Then $u_i$ is constant in the interval $[\beta, \infty)$, which is non-decreasing and concave and includes the interval in the definition. Using these utilities we recover the standard objective to maximize the number of successful transmissions, where a success is a transmission attempt of a link with SINR above $\beta$.
- For link-weighted capacity maximization we use the same $(c, \beta)$ and $u_i(x) = w_i \geq 0$ for all $x \geq \beta$. This yields optimization of successful transmissions, where the successes of link $i$ are weighted by $w_i$.
- For utilities representing Shannon capacity we assume $u_i(\gamma_i^R) = \log(1 + \gamma_i^R)$ throughout. This function is non-decreasing and concave over the whole interval $[0, \infty)$ and thereby directly fulfills the definition. This results in maximizing the total Shannon capacity of the network.

Approximation guarantees for algorithms in the non-fading model usually rely on signal strengths $\bar{S_{j,i}}$ being tied to an underlying geometry of link nodes. For example, a popular assumption is $\bar{S_{j,i}} = p_j / d(s_j, r_i)^\alpha$, where $p_j$ is the transmission power and $d(s_j, r_i)$ the distance between $s_j$ and $r_i$. In contrast, our connection between Rayleigh-fading and non-fading models shown below applies in a more general scenario, without any assumptions on the values of the (expected) signal strength $\bar{S_{j,i}}$ – except non-negativity and the relation to noise as detailed above. In particular, this implies that our reduction between the models holds for arbitrary power assignments, path-loss exponents, requests located in metric spaces, etc. For proving bounded approximation factors, however, algorithms for the non-fading model usually rely heavily on $\bar{S_{j,i}}$ being characterized by these parameters. Consequently, our "black-box" translation of these algorithms and their approximation factors also applies only to instances of the Rayleigh model that have expected values $\bar{S_{j,i}}$ with the same characteristics.

# 3 Probabilities in the Rayleigh-fading model

In this section, we consider the following situation under Rayleigh-fading constraints. Assuming each sender $s_i$ transmits with probability $q_i$, we bound the probability to reach an SINR above a certain threshold $\beta$. We denote this probability by $Q_i(q_1, \ldots, q_n, \beta)$. Fortunately, in contrast to the non-fading model, the success probability can be given in a closed-form expression.

**Theorem 1.** *The probability that receiver $r_i$ receives the signal from $s_i$ with at least an SINR of $\beta$ is*

$$Q_i(q_1, \ldots, q_n, \beta) = q_i \cdot \exp\left(-\frac{\beta\nu}{\bar{S_{i,i}}}\right) \prod_{j \neq i} \left(1 - \frac{\beta q_j}{\beta + \frac{\bar{S_{i,i}}}{\bar{S_{j,i}}}}\right) .$$

The proof of this expression is mainly due to Liu and Haenggi [18]; it can be found in the appendix. The expression has the advantage of being an exact probability. However, in order to compare the probability to the one in the non-fading channel model, we need upper and lower bounds.

**Lemma 1.** *The probability for link $i$ to reach an SINR of $\beta$ is at least*

$$Q_i(q_1, \ldots, q_n, \beta) \geq q_i \cdot \exp\left(-\frac{\beta}{\bar{S_{i,i}}}\left(\nu + \sum_{j \neq i} \bar{S_{j,i}} q_j\right)\right) .$$

*The probability for link $i$ to reach an SINR of $\beta$ is at most*

$$\begin{aligned} &Q_i(q_1, \ldots, q_n, \beta) \\ &\leq q_i \cdot \exp\left(-\frac{\beta\nu}{\bar{S_{i,i}}} - \sum_{j \neq i} \min\left\{\frac{1}{2}, \frac{\beta \bar{S_{j,i}}}{2\bar{S_{i,i}}}\right\} q_j\right) . \end{aligned}$$

*Proof:* The proof of this lemma is based on the following observation concerning the exponential function.

**Observation 1.** *For all $x \in \mathbb{R}$, $q \in [0, 1]$, we have*

$$\exp(-xq) \leq 1 - \frac{q}{\frac{1}{x} + 1} ,$$

*and for all $x \in (0, 1]$, $q \in [0, 1]$, we have*

$$1 - \frac{q}{\frac{1}{x} + 1} \leq \exp\left(-\frac{1}{2}xq\right) .$$

*Proof:* We show the first inequality using the fact that $\exp(y) \geq 1 + y$ for all $y \in \mathbb{R}$. Setting $y = xq$ yields

$$\exp(-xq) = \frac{1}{\exp(xq)} \leq \frac{1}{1+xq} = 1 - \frac{q}{\frac{1}{x}+q} \leq 1 - \frac{q}{\frac{1}{x}+1} \ .$$

Setting $y = -\frac{q}{\frac{1}{x}+1}$, we get

$$1 - \frac{q}{\frac{1}{x}+1} \leq \exp\left(-\frac{q}{\frac{1}{x}+1}\right) = \exp\left(-\frac{xq}{1+x}\right) \ .$$

Furthermore, we have for all $x \in (0,1]$ that $\frac{x}{x+1} \geq \frac{1}{2}x$. This yields the second bound. □

Setting now $q = q_j$ and $x = \beta \bar{S_{j,i}} / \bar{S_{i,i}}$ in this observation, we get

$$\exp\left(-\beta \frac{\bar{S_{j,i}}}{\bar{S_{i,i}}} q_j\right) \leq 1 - \frac{\beta q_j}{\beta + \bar{S_{i,i}}/\bar{S_{j,i}}} \ ,$$

and setting $x = \min\left\{1, \frac{\beta \bar{S_{j,i}}}{\bar{S_{i,i}}}\right\}$ yields

$$1 - \frac{\beta q_j}{\beta + \bar{S_{i,i}}/\bar{S_{j,i}}} \leq \exp\left(-\frac{1}{2}\min\left\{1, \frac{\beta \bar{S_{j,i}}}{\bar{S_{i,i}}}\right\} q_j\right) \ .$$

Theorem 1 now yields the claim. □

## 4 Transforming Scheduling Algorithms

The bounds given in the previous section immediately allow us to estimate the performance of algorithms for the non-fading model in a Rayleigh-fading environment after some minor modifications.

In particular, we can take an arbitrary approximation algorithm for capacity maximization. For example, we can use the algorithm that achieves an $O(\log n)$-approximation [22] for the case of general utility functions. If utility functions are threshold functions, we might also use the constant-factor approximations for the setting with uniform transmission powers [8] or monotone transmission powers [7], or even for the case in which the algorithm has to choose the transmission power itself [6]. In any case, making exactly the links transmit with probability 1 (without changes of the transmission powers), Lemma 2 yields that the expected utility under Rayleigh-fading interference is at least a $1/\mathrm{e}$-fraction of the utility in the non-fading model.

**Lemma 2.** *Consider the solution for capacity maximization in the non-fading model. Let the SINR that link i reaches in this solution be denoted by $\gamma_i^{nf}$. Making the same senders transmit with the same transmission powers, we have*

$$\mathbf{E}\left[\sum_i u_i\left(\gamma_i^R\right)\right] \geq \frac{1}{\mathrm{e}} \sum_i u_i\left(\gamma_i^{nf}\right) \ .$$

*Proof:* Having a solution for capacity maximization in the non-fading model, we can simply transfer this solution to the fading model by making the same senders transmit. That is, we set $q_i = 1$ if link $i$ transmits in the non-fading solution and $q_i = 0$ otherwise.

Let $S \subseteq [n]$ be the set of links transmitting in the non-fading solution. The SINR of link $i$ in the non-fading model is given by

$$\gamma_i^{\mathrm{nf}} = \frac{\bar{S_{i,i}}}{\sum_{j \in S} \bar{S_{j,i}} + \nu} \ .$$

Using Lemma 1, we can deduce that in the Rayleigh-fading model $Q_i(q_1, \ldots, q_n, \gamma_i^{\mathrm{nf}}) \geq \frac{1}{\mathrm{e}}$ for all $i \in S$.

In combination, this means that for the resulting algorithm

$$\mathbf{E}\left[u_i\left(\gamma_i^R\right)\right] \geq u_i\left(\gamma_i^{\mathrm{nf}}\right) \cdot Q_i(q_1, \ldots, q_n, \gamma_i^{\mathrm{nf}}) \geq u_i\left(\gamma_i^{\mathrm{nf}}\right) \cdot \frac{1}{\mathrm{e}} \ .$$

This directly proves the lemma by summing over all links $i$. □

In terms of our objective function "capacity" this means that we are at most a $1/\mathrm{e}$-factor worse in expectation. In combination, this means that the resulting algorithm will compute transmission probabilities yielding an expected capacity that is at most a constant factor worse than the optimally achievable capacity in the non-fading model. However, it remains to show that the theoretical optimum in the Rayleigh-fading model cannot be much better than the one in the non-fading model. This will be carried out in Section 5.

Existing approximation algorithms to minimize latency can in general be divided into two classes. On the one hand, there are many algorithms actually attempting to maximize the utilization of the first time slot and then apply this procedure recursively on the remaining links. For these kinds of algorithms and analyses exactly the same argumentation as for capacity maximization can be applied. On the other hand, ALOHA-style protocols have been proposed. Here, in each time slot, each link is assigned a (small) transmission probability, which we assume to be smaller than $1/2$. If it is successful, the sender stops transmitting, otherwise it continues running the algorithm. In order to transform such algorithms to the Rayleigh-fading model, we let each (randomized) step be executed 4 times. Due to repeating those independent random choices, we can increase the probability that at least one of the 4 transmissions reaches an SINR of $\beta$. This yields a success probability that is at least as large as in the non-fading model. If $p$ is the probability to reach a certain SINR threshold $\beta$ in the non-fading model, Lemma 1 yields this probability for the Rayleigh-fading model being at least $p \cdot 1/\mathrm{e}$. In 4 independent repeats the probability of reaching $\beta$ at least once is therefore at least $1 - (1 - p/\mathrm{e})^4$. This is at least $p$ if the transmission probability (and therefore the success probability) is at most $1/2$.

For multi-hop scheduling algorithms [6], [9], the single-hop transformations mentioned above can directly be generalized. In this setting a transmission can be sent via intermediate nodes forwarding the respective data. Here, in fact, the resulting multi-hop schedule can also be considered as a concatenation of single-hop schedules. Transforming each of them in the described way, we still only lose constant factors. More precisely speaking, those multi-hop scheduling algorithms yield a schedule for every single time slot. Using the transformation on every single schedule (and possibly executing each step 4 times) yields the bounds on the utility and the success probability of a transmission discussed above for every such schedule. In total, the multiple schedules yield the same constant factors discussed above in terms of utility or success.

# 5 Transforming the Rayleigh-Fading Optimum

The performance of all algorithms constructed in Section 4 were measured in terms of the value of the optimal solution in the non-fading model. However, in order to derive approximation guarantees for the Rayleigh-fading model, the value of the computed solution has to be compared within the Rayleigh-fading model. Here, the optimal solution could potentially be much better than the non-fading one. In this section, we give a possibly surprising result that this indeed cannot happen.

More formally, we will show that for both capacity maximization and latency minimization the Rayleigh-fading optimum can be at most an $O(\log^* n)$-factor better than the non-fading optimum, where $\log^*$ is the iterated logarithm. This being a small number even for quite large $n$ yields basically a constant factor in practical terms. Thus, in settings with realistic network size the difference between the non-fading optimum and the Rayleigh-fading optimum can be considered almost constant.

**Theorem 2.** *Given an assignment of transmission probabilities $q_1, \ldots, q_n$, there is a (potentially different) assignment of transmission probabilities $q'_1, \ldots, q'_n$ such that*

$$\mathbf{E}\left[\sum_i u_i\left(\gamma_i^{nf}\right)\right] \geq \Omega\left(\frac{1}{\log^* n}\right)\mathbf{E}\left[\sum_i u_i\left(\gamma_i^{R}\right)\right] \quad ,$$

*where $\gamma_i^{nf}$ is the non-fading SINR achieved by link $i$ when using transmission probabilities $q'_1, \ldots, q'_n$.*

*Proof:* To show the theorem, we "simulate" the single random trial with $O(\log^* n)$ independent steps in the non-fading model, in which different assignments of transmission probabilities are used. We will come to the conclusion that the expected sum of utilities a link achieves in all simulation steps combined is at least as large as in the single Rayleigh-fading step divided by a constant. Therefore, taking the best one of these steps shows the theorem.

To focus on the main argument, we assume that $c_i \geq 3$ for all $i$. The general proof idea, however, is applicable for any constants $c_i$ after only modifying the involved constants. We define the sequence $(b_k)_{k\in\mathbb{N}}$ recursively by setting $b_0 = 1/4$, $b_{k+1} = \exp(b_k/2)$. The simulation works as follows. For each $k \geq 0$ with $b_k < n$, we let each sender transmit with probability $q_i^{(k)} := q_i/4b_k$ for 19 times independently at random. As $(b_k)_{k\in\mathbb{N}}$ is an iterated exponential sequence, $b_k \geq n$ for some $k = O(\log^* n)$. This means that we make $O(\log^* n)$ transmission attempts in total.

**Algorithm 1:** Formal description of the simulation.

$k := 0, b_0 := 1/4$;
**while** $b_k < n$ **do**
  **for** 19 *times* **do**
    transmit with probability $q_i^{(k)} := q_i/4b_k$;
  $b_{k+1} := \exp(b_k/2), k := k+1$;

Let us first turn to the case of low SINR values. These are mainly determined by interference. We achieve the following probability bound.

**Lemma 3.** *For each link $i \in [n]$, the probability to reach any fixed SINR thresholds $\beta \leq \frac{\bar{S}_{i,i}}{2\nu}$ in the non-fading model in at least one of the simulation steps is at least $Q_i(q_1, \ldots, q_n, \beta)$ for each link $i$.*

*Proof:* Consider an arbitrary $i \in [n]$. We claim: The probability of achieving SINR of at least $\beta$ in the non-fading model during one of the $O(\log^* n)$ repeats is at least $Q_i(q_1, \ldots, q_n, \beta)$.

We set $A_i = \sum_{j\neq i} \min\left\{1, \beta \bar{S}_{j,i}/\bar{S}_{i,i}\right\} \cdot q_j$. Observe that $0 \leq A_i \leq n$. In order to bound the success probability, we only take the $k$th iteration of the *while* loop into account, where $b_k \leq \exp(A_i/2) \leq \exp(b_k/2)$. We will show that in this iteration, the probability of a successful transmission in the non-fading model is at least as large as the original one in the Rayleigh-fading model. Lemma 1 yields $Q_i(q_1, \ldots, q_n) \leq q_i \cdot \exp\left(-\frac{\beta\nu}{\bar{S}_{i,i}} - \frac{A_i}{2}\right)$. Using this, we observe that the probability of success in the Rayleigh-fading model is at most $\frac{q_i}{e^{A_i/2}} \leq \frac{q_i}{b_k}$.

Let us first consider a single one of the 19 independent iterations. Let $X_j$ be a 0/1 random variable indicating if sender $s_j$ transmits in this iteration. By definition $\mathbf{E}\left[X_j\right] = q_j^{(k)}$. Furthermore, set $Z_i = \sum_{j\neq i} \min\left\{1, \beta \bar{S}_{j,i}/\bar{S}_{i,i}\right\} \cdot X_j$.

To make the transmission successful in the non-fading model, we have to have $X_i = 1$ and $\bar{S}_{i,i} \geq \beta(\sum_{j\neq i} \bar{S}_{j,i} X_j + \nu)$. To bound the probability of the latter event, we use the assumption that $\bar{S}_{i,i} \geq 2\beta\nu$. With this, we get $\beta(\sum_{j\neq i} \bar{S}_{j,i}/\bar{S}_{i,i} X_j + \nu/\bar{S}_{i,i}) \leq Z_i + 1/2$. Therefore it suffices to have $Z < 1/2$ for this to yield $\bar{S}_{i,i} \geq \beta(\sum_{j\neq i} \bar{S}_{j,i} X_j + \nu)$.

This allows to estimate the probability of this event by Markov's inequality using

$$\mathbf{Pr}\left[Z_i \geq \frac{1}{2}\right] \leq 2\mathbf{E}\left[Z_i\right] = 2\sum_{j\neq i} \min\left\{1, \beta\frac{\bar{S}_{j,i}}{\bar{S}_{i,i}}\right\} \mathbf{E}\left[X_j\right]$$
$$= 2\sum_{j\neq i} \min\left\{1, \beta\frac{\bar{S}_{j,i}}{\bar{S}_{i,i}}\right\} \cdot \frac{q_j}{4b_k} \leq 2\frac{A_i}{4b_k} \quad . \tag{1}$$

For the remaining considerations, we distinguish between the two cases $k = 0$ and $k \geq 1$.

In the case $k \geq 1$, we use the fact that $A_i \leq b_k$ to get that the success probability in the non-fading model in a single iteration is at least

$$q_i^{(k)} \cdot \left(1 - 2\frac{A_i}{4b_k}\right) \geq \frac{q_i^{(k)}}{2} = \frac{q_i}{8b_k} \quad .$$

We use now the facts that $k \geq 1$ and therefore $b_k \geq \exp(1/8)$ and furthermore that for all $0 \leq x \leq \exp(-1/8)$ we have $1 - (1 - x/8)^{19} \geq x$. We can easily derive this claim by rearranging the inequality to $(1-x)^{1/19} \geq 1 - x/8$. The lefthand side is concave and monotone decreasing in $[0, 1]$. Thus, $(1-x)^{1/19}$ and the linear function $1 - x/8$ can be equal in at most 2 points. One such intersection is $x = 0$. The other one is between $x = \exp(1/8)$ and $x = 1$ as $(1-x)^{1/19} = 0 < 1 - x/8$ for $x = 1$ and $(1-x)^{1/19} > 1 - x/8$ for $x = \exp(1/8)$. Due to continuity of the functions, this proves the inequality stated above.

Having $q_i/b_k \leq 1/b_k \leq \exp(-1/8)$ yields that in 19 independent repeats, we get a total success probability of at least

$$1 - \left(1 - \frac{q_i}{8b_k}\right)^{19} \geq \frac{q_i}{b_k} \geq q_i \exp\left(-\frac{A_i}{2}\right) \quad .$$

As we have already seen, the success probability in the Rayleigh-fading model is at most $q_i \exp(-A_i/2)$.

For the case $k = 0$, we use that the probability that the transmission is not successful within a single iteration of the inner loop is at most $q_i(1 - 2A_i)$. This results from Equation 1, which bounds the probability for an unsuccessful transmission assuming $i$ transmits by $1 - 2A_i$ for $k = 0$. The probability that at least one of the 19 independent repeats is successful is at least $1 - (1 - q_i(1 - 2A_i))^{19} \geq q_i \exp(-A_i/2)$ for all $0 \leq q_i \leq 1$ because $A_i \leq 1/4$. □

Now, let $\gamma_i^{\mathrm{nf},t}$ be the SINR that is achieved in the $t$th transmission attempt of sender $i$, i.e., in the $t$th overall iteration of the *for* loop. In this notation, Lemma 3 shows $\mathbf{Pr}\left[\max_t \gamma_i^{\mathrm{nf},t} \geq \beta\right] \geq \mathbf{Pr}\left[\gamma_i^R \geq \beta\right]$ for all $\beta \leq \frac{\bar{S}_{i,i}}{2\nu}$.

To show the theorem, we need to upper-bound $\mathbf{E}\left[u_i(\gamma_i^R)\right]$ in terms of $\mathbf{E}\left[u_i(\max_t \gamma_i^{\mathrm{nf},t})\right]$. For this purpose, we use the following decomposition:

$$\begin{aligned}\mathbf{E}\left[u_i(\gamma_i^R)\right] = \mathbf{Pr}\left[\gamma_i^R < \frac{\bar{S}_{i,i}}{2\nu}\right] \mathbf{E}\left[u_i(\gamma_i^R) \;\middle|\; \gamma_i^R < \frac{\bar{S}_{i,i}}{2\nu}\right] \\ + \mathbf{Pr}\left[\gamma_i^R \geq \frac{\bar{S}_{i,i}}{2\nu}\right] \mathbf{E}\left[u_i(\gamma_i^R) \;\middle|\; \gamma_i^R \geq \frac{\bar{S}_{i,i}}{2\nu}\right] \quad .\end{aligned}$$

Defining $\tilde{u}_i$ by $\tilde{u}_i(\gamma) = u_i(\gamma)$ for $\gamma \leq \frac{\bar{S}_{i,i}}{2\nu}$ and $\tilde{u}_i(\gamma) = u_i(\frac{\bar{S}_{i,i}}{2\nu})$ for $\gamma > \frac{\bar{S}_{i,i}}{2\nu}$, we have

$$\begin{aligned}&\mathbf{Pr}\left[\gamma_i^R < \frac{\bar{S}_{i,i}}{2\nu}\right] \mathbf{E}\left[u_i(\gamma_i^R) \;\middle|\; \gamma_i^R < \frac{\bar{S}_{i,i}}{2\nu}\right] \\ &= \mathbf{Pr}\left[\gamma_i^R < \frac{\bar{S}_{i,i}}{2\nu}\right] \mathbf{E}\left[\tilde{u}_i(\gamma_i^R) \;\middle|\; \gamma_i^R < \frac{\bar{S}_{i,i}}{2\nu}\right] \\ &\leq \mathbf{E}\left[\tilde{u}_i(\gamma_i^R)\right] \leq \mathbf{E}\left[u_i(\max_t \gamma_i^{\mathrm{nf},t})\right] \quad ,\end{aligned}$$

where the last step is due to Lemma 3. Furthermore, by concavity of $u_i$ on $\left[\frac{\bar{S}_{i,i}}{3\nu}, \infty\right)$, we have

$$\mathbf{E}\left[u_i(\gamma_i^R) \;\middle|\; \gamma_i^R \geq \frac{\bar{S}_{i,i}}{2\nu}\right] \leq u_i\left(\mathbf{E}\left[\gamma_i^R \;\middle|\; \gamma_i^R \geq \frac{\bar{S}_{i,i}}{2\nu}\right]\right) \quad .$$

We can bound $\mathbf{E}\left[\gamma_i^R \;\middle|\; \gamma_i^R \geq \frac{\bar{S}_{i,i}}{2\nu}\right]$ by using that for any fixed $S_{j,i}$ we have

$$\begin{aligned}&\mathbf{E}\left[\gamma_i^R \;\middle|\; \gamma_i^R \geq \frac{\bar{S}_{i,i}}{2\nu} \text{ and fixed } S_{j,i} \text{ for } j \neq i\right] \\ &= \frac{1}{\sum_{j \neq i} S_{j,i} + \nu} \mathbf{E}\left[S_{i,i} \;\middle|\; S_{i,i} \geq \frac{\bar{S}_{i,i}}{2\nu}\left(\sum_{j \neq i} S_{j,i} + \nu\right)\right] \quad .\end{aligned}$$

As $S_{i,i}$ is exponentially distributed, this term is equal to

$$\begin{aligned}&\frac{1}{\sum_{j \neq i} S_{j,i} + \nu}\left(\bar{S}_{i,i} + \frac{\bar{S}_{i,i}}{2\nu}\left(\sum_{j \neq i} S_{j,i} + \nu\right)\right) \\ &= \frac{1}{\sum_{j \neq i} S_{j,i} + \nu}\bar{S}_{i,i} + \frac{\bar{S}_{i,i}}{2\nu} \\ &\leq \frac{\bar{S}_{i,i}}{\nu} + \frac{\bar{S}_{i,i}}{2\nu} \\ &\leq \frac{3\bar{S}_{i,i}}{2\nu} \quad .\end{aligned}$$

For any concave function $f(x)$, it holds $6/7 \cdot f(x/3) + 1/7 \cdot f(3x/2) \leq f(6x/21 + 3x/14) = f(x/2)$. Thus, using concavity once again, we get $u_i\left(\frac{\bar{S}_{i,i}}{2\nu}\right) \geq \frac{6}{7}u_i\left(\frac{\bar{S}_{i,i}}{3\nu}\right) + \frac{1}{7}u_i\left(\frac{3\bar{S}_{i,i}}{2\nu}\right) \geq \frac{1}{7}u_i\left(\frac{3\bar{S}_{i,i}}{2\nu}\right)$ yielding

$$u_i\left(\mathbf{E}\left[\gamma_i^R \;\middle|\; \gamma_i^R > \frac{\bar{S}_{i,i}}{2\nu}\right]\right) \leq u_i\left(\frac{3\bar{S}_{i,i}}{2\nu}\right) \leq 7u_i\left(\frac{\bar{S}_{i,i}}{2\nu}\right) \quad .$$

Using Lemma 3 another time, we get $\mathbf{Pr}\left[\gamma_i^R \geq \frac{\bar{S}_{i,i}}{2\nu}\right] \leq \mathbf{Pr}\left[\max_t \gamma_i^{\mathrm{nf},t} \geq \frac{\bar{S}_{i,i}}{2\nu}\right]$ and this yields

$$\mathbf{Pr}\left[\gamma_i^R \geq \frac{\bar{S}_{i,i}}{2\nu}\right] u_i\left(\frac{\bar{S}_{i,i}}{2\nu}\right) \leq \mathbf{E}\left[u_i(\max_t \gamma_i^{\mathrm{nf},t})\right] \quad .$$

In combination, we get

$$\mathbf{E}\left[u_i(\gamma_i^R)\right] \leq 8\mathbf{E}\left[u_i(\max_t \gamma_i^{\mathrm{nf},t})\right]$$

This completes the proof. □

This way, we see that we lose at most an $O(\log^* n)$ factor in all approximation guarantees of non-fading algorithms. In particular, the constant-factor capacity-maximization algorithms of the non-fading case provide without any further modification $O(\log^* n)$ approximations in the Rayleigh-fading case.

When considering latency minimization under Rayleigh-fading conditions, the optimum should rather be considered as an algorithm itself that assigns different transmission probabilities in each step, because the optimum consists of a different schedule in every time step. This assignment may arbitrarily depend on previous successes and may be computed using arbitrary computation power. However, Theorem 2 shows for this case that even the perfect algorithm computes schedules that are at most an $O(\log^* n)$ factor shorter than the non-fading optimum, because we could replace each times lot by the described simulation, increasing the schedule length by a factor of at most $O(\log^* n)$.

## 6 Regret Learning for Capacity Maximization

Most of the existing algorithms for capacity maximization are centralized. A notable exception is the round-based distributed approach presented by Dinitz [11] using regret-learning techniques. This approach works for binary utilities with a global threshold $\beta$ (c.f. Section 2), and we assume that we have binary utilities throughout this and the next section. Thus, we focus on the number of *successful transmissions*, which are all transmission attempts of a link that achieve an SINR above $\beta$. In addition, we use the notion of a *feasible set*, which is a subset of links such that they can all simultaneously transmit successfully.

The idea behind the regret-learning approach is that a sequence of action vectors is computed in a decentralized way. In each step $t$, every user $i$ decides which action $a_i^{(t)}$ to take. Together those chosen actions form an *action vector* $a^{(t)}$. After taking the action, he gets a reward $h_i(a_1^{(t)}, \ldots, a_n^{(t)})$ depending on his own choice and the one of the other users in step $t$. The choice of which action to choose then depends on the history of rewards experienced before. The *external regret* is defined as the difference between the reward of the best single action in hindsight and the summed rewards experienced by the algorithm.

**Definition 2.** *The* (external) regret *of user $i$ at time $T$ given a sequence of action vectors $a^{(1)}, \ldots, a^{(T)}$ is*

$$\max_{a'_i \in \mathcal{A}_i} \sum_{t=1}^{T} h_i(a_1^{(t)}, \ldots, a'_i, \ldots, a_n^{(t)}) - \sum_{t=1}^{T} h_i(a^{(t)}) \enspace ,$$

*where $\mathcal{A}_i$ is the set of possible actions of user $i$.*

The user regrets what he might have won by switching to one single action for all time steps in hindsight instead of using the algorithm. An algorithm has the no-regret property if the average regret per time step converges to $0$ for the number of time steps $T$ going to $\infty$. Similar to previous work, our approach relies on algorithms that achieve the no-regret property for a single user with high probability after a number of steps polynomial in $n$. Such algorithms are known in the literature [23], [24].

To adapt this framework to capacity maximization, we assume that each link $i$ is a user who has two actions in each step $t$ – namely to send ($q_i = 1$) or not to send ($q_i = 0$). The reward function $h_i(q_1, \ldots, q_n)$ is set to be $1$ when a user $i$ sends and is successful (SINR above $\beta$), to be $-1$ when user $i$ sends and is not successful, and $0$ if a user does not send.

Using this setup, Ásgeirsson and Mitra [24] showed for the non-fading model that if each user applies a no-regret algorithm the average number of successful transmissions over time converges to the optimum up to a constant factor. This result relies on distance-based interferences and uniform transmission powers. More formally, the result applies when $\bar{S}_{j,i} = p_j / d(s_j, r_i)^\alpha$, where $p_j = 1$ is the transmission power and $d(s_j, r_i)$ the distance between $s_j$ and $r_i$ in some metric space.

Unfortunately, our black-box transformation cannot be directly applied here. This is due to the sequential nature of the algorithm: In each round, a user's reaction depends on the fact if transmissions were successful in earlier rounds. As success of transmissions is now stochastic, the effects on the dynamic algorithm are not immediately clear. Luckily, we can nevertheless reproduce the necessary results without altering the regret-learning algorithms. Thus, we are able to prove a similar result showing that the expected number of successful transmissions in the Rayleigh model converges to the non-fading optimum up to a constant factor. In particular, we prove the following theorem.

**Theorem 3.** *If all users apply no-regret algorithms with respect to rewards functions $h_i$, the average number of successful transmissions per round converges to $\Omega(|\mathrm{OPT}|)$, for* $\mathrm{OPT}$ *being a largest feasible set in the non-fading model with distance-based interference and uniform transmission powers.*

In the Rayleigh-fading model, the reward function itself is stochastic. However, as we will see, any no-regret algorithm with respect to the stochastic reward functions $h_i$ is also has the no-regret property with respect to the (deterministic) reward functions $\bar{h}_i$, which are defined as the respective expectation of $h_i$. To define $\bar{h}_i$ formally, let us recall the possible outcomes of $h_i$. Whenever a link $i$ does not try to transmit, it gets a reward of $0$. In time steps with transmission attempt, link $i$ gets a reward of $1$ if the transmission is successful for link $i$. This happens with probability $Q_i(q_1, \ldots, q_n, \beta)$. Otherwise the reward is $-1$. In total the expected reward of a time step with transmission attempt of link $i$ evaluates to $2 \cdot Q_i(q_1, \ldots, q_n, \beta) - 1$. So, the expected reward of user $i$ is

$$\bar{h}_i(q_1, \ldots, q_n) = \mathbf{E}\left[h_i(q_1, \ldots, q_n)\right] = \begin{cases} 0 & \text{if } q_i = 0, \\ 2 \cdot Q_i(q_1, \ldots, q_n, \beta) - 1 & \text{if } q_i = 1. \end{cases}$$

We approach the proof of Theorem 3 as follows. Consider a joint run of the no-regret algorithms for all users for $T$ rounds in hindsight. Assume that the regret for some user $i$ with respect to $h_i$ is $\epsilon' \cdot T$ for a constant $\epsilon' < 1$. Note that each user runs an algorithm that has this property for him individually with probability $(1 - 1/\Omega(n)))$ after a number of rounds $T$ that is polynomial in $n$, and $1/\epsilon'$. Under the assumption that a user has regret $\epsilon' \cdot T$ for $h_i$, we show in the subsequent Lemma 4 that with probability at least $(1 - 1/T^2)$ the regret for user $i$ of the same action sequence against rewards given by $\bar{h}_i$ is at most $\epsilon \cdot T$, where $\epsilon < 1$ becomes arbitrarily small when $T$ gets sufficiently large (but polynomial in $n$ and $1/\epsilon$). Hence, as $T > n$, with probability at least $(1 - 1/\Omega(n))$, a single user $i$ has regret $\epsilon \cdot T$ for $h_i$. Thus, using a union bound, with constant probability this condition holds for all users simultaneously. For sequences that give all users low regret simultaneously, we show in Theorem 4 that the throughput can be related to the optimum, which directly implies the constant factor in Theorem 3.

**Lemma 4.** *Fix some user $i \in [n]$ running a no-regret algorithm with respect to $h_i$ and some round $T \in \mathbb{N}$. Let $R_h$ and $R_{\bar{h}}$ be the user's regret with respect to reward functions $h_i$ and $\bar{h}_i$, respectively. Then with probability at least $(1 - 1/T^2)$ we have $R_{\bar{h}} \leq R_h + O(\sqrt{T \ln T})$.*

*Proof:* For $a \in \{0, 1\}$, $t \in [T]$, let us define the random variable $Y_t^a$ by

$$Y_t^a = \left(\bar{h}_i(a, q_{-i}^{(t)}) - \bar{h}_i(q^{(t)})\right) - \left(h_i(a, q_{-i}^{(t)}) - h_i(q^{(t)})\right) \quad .$$

Observe that $-2 \leq Y_t^a \leq 2$ and that, conditioned on any outcomes of $Y_1^a, \ldots, Y_{t-1}^a$, the expectation is always 0. Therefore, we can apply Hoeffding's inequality to get

$$\mathbf{Pr}\left[\sum_{t=1}^{T} Y_t^a \geq \sqrt{16 T \ln T}\right] \leq \exp\left(-\frac{2(16 T \ln T)}{16T}\right) = \frac{1}{T^2}$$

So, with probability at most $\frac{1}{T^2}$, we have $\sum_{t=1}^{T} Y_t^a \geq \sqrt{16 T \ln T}$ for $a = 0$ and $a = 1$. In return, this implies, $R_{\bar{h}} \leq R_h + \sqrt{16 T \ln T}$ with probability at least $(1 - 1/T^2)$. □

Based on Lemma 4, we now assume that we have a fixed sequence of length $T$ in which each user's regret with respect to the expected rewards $\bar{h}_i$ is at most $\epsilon \cdot T$. We will show that in this sequence, on average per round, the expected number of successful transmissions is $\Omega(|\mathrm{OPT}|)$.

**Theorem 4.** *Consider a sequence $q^{(1)}, \ldots, q^{(T)}$ of action vectors such that each user $i$ has regret at most $\epsilon \cdot T$ with respect to reward function $\bar{h}_i$. Then the average number of successful transmissions over the $T$ time steps is in $\Omega(|\mathrm{OPT}| - \epsilon n)$,*

Theorem 4 directly follows from Lemma 5 and Lemma 6, which we will prove in the remaining part of this section. For any no-regret algorithm we get $\epsilon$ converging to 0 after a sufficient amount of time steps. To be precise, when $\epsilon < 1/n$ we can guarantee a factor in $\Omega(|\mathrm{OPT}|)$. We assume here that the no-regret algorithm converges in a polynomial fashion (depending on the number of available actions) yielding this guarantee with high probability after a time polynomial in $n$.

Note that this theorem together with Theorem 2 yields a factor of $O(\log^* n)$ in comparison to the Rayleigh-fading optimum. Our analysis extends the one for the non-fading case [11], [24]. The results from [24] also show that the number of successful transmissions is bound in $\Omega(|\mathrm{OPT}|)$ for regret learning in the non-fading model. This highlights the close relationship between the models.

In the following, we consider a sequence $q^{(1)}, \ldots, q^{(T)}$ that exhibits external regret $\epsilon \cdot T$ for every user $i = 1, \ldots, n$. We define $f_i = \frac{1}{T}\sum_t q_i^{(t)}$ as the fraction of time steps the user chooses $q_i = 1$. Let $F = \sum_i f_i$. We define $x_i$ to be the average success probability per time step with $x_i = \frac{1}{T}\sum_t Q_i^{(t)}\left(q_1^{(t)}, \ldots, q_n^{(t)}, \beta\right)$, and we set $X = \sum_i x_i$. That is, $X$ is the expected number of successful transmissions on average per round.

We examine such sequences and at first bound the expected number of successful transmissions on average per round. It turns out that for $\epsilon$ approaching 0 half of the transmissions are successful in the long run. Besides this result, we will show that the average number of transmitting nodes $F$ is in $\Omega(|\mathrm{OPT}|)$. This together shows that the expected number of successful transmissions $X$ on average per round is in $\Omega(|\mathrm{OPT}|)$.

**Lemma 5.** $X \leq F \leq 2X + \epsilon n$

*Proof:* The first inequality follows by definition. For the second inequality, we use the fact that for each user $i$ the regret is at most $\epsilon$. Therefore, always using action $q_i = 0$ can increase the average reward per step by at most $\epsilon$. Formally this means $2 \cdot x_i - f_i \geq -\epsilon$. Taking the sum over all $i$, we get $2X - F \geq -\epsilon n$. This yields $F \leq 2X + \epsilon n$. □

As we have seen that $X = \Theta(F)$, it suffices now to show that $F$ is in $\Omega(|\mathrm{OPT}|)$.

**Lemma 6.** *Let* OPT *denote a largest feasible set in the non-fading model under uniform transmission powers, then $F = \Omega(|\mathrm{OPT}|)$.*

*Proof:* In the following $a(j, i)$ denotes the affectance defined as follows (cf. the definition in [25]). The affectance of link $j$ on link $i$ for uniform powers is

$$a(j,i) = \min\left\{1, \frac{\beta \cdot \frac{d(s_i,r_i)^\alpha}{d(s_j,r_i)^\alpha}}{1 - \beta \cdot \nu \cdot d\left(s_i, r_i\right)^\alpha}\right\} \quad .$$

We will denote the summed affectance from other links on link $i$ by

$$a^{(t)}(i) = \sum_{\substack{j \in [n] \\ q_j^{(t)} = 1}} a(j, i) \quad .$$

Note that by definition for a link $i$ the SINR constraint is fulfilled iff $a^{(t)}(i) \leq 1$.

Let $P_i$ be the fraction of steps in which $a^{(t)}(i) \leq \frac{1}{2}$ and let $\hat{a}(i) = \frac{1}{T}\sum_t a^{(t)}(i)$. We define the sets $\mathrm{OPT}' = \left\{i \in \mathrm{OPT} \colon f_i < \frac{1}{2} - \epsilon\right\}$ and $\mathrm{OPT}'' = \left\{i \in \mathrm{OPT}' \colon \sum_{j \in \mathrm{OPT}'} a(i,j) \leq 2\right\}$. So all links in $\mathrm{OPT}''$ attempt to transmit in less than a $\frac{1}{2} - \epsilon$ fraction of the time and affect others doing so by at most 2.

If $|\mathrm{OPT} \setminus \mathrm{OPT}'| > |\mathrm{OPT}|/2$, then $F$ would be at least $\left(\frac{1}{2} - \epsilon\right) \cdot |\mathrm{OPT} \setminus \mathrm{OPT}'|$ and therefore in $\Omega(|\mathrm{OPT}|)$.

So we consider $|\mathrm{OPT}'| \geq |\mathrm{OPT}|/2$ for the rest of the proof. The choice of $\mathrm{OPT}'$ directly corresponds to the choice of $L'$ in [24, Lemma 8] stated as follows.

**Lemma 7** (Lemma 8 in [24])**.** *Let $L$ be a feasible set. Define the set $L' = \left\{u \in L \;\middle|\; \sum_{v \in L} a(u,v) \leq 2\right\}$. Then $|L'| \geq |L|/2$.*

Using this, we see $|\mathrm{OPT}''| \geq |\mathrm{OPT}|/4$. Therefore, it is sufficient to show $F = \Omega(|\mathrm{OPT}''|)$ and so we just need to consider links $i \in \mathrm{OPT}''$.

We consider the reward gain for link $i$ by switching to action $q_i = 1$ throughout every step. In an $f_i$ fraction of the steps nothing changes. In at least a $P_i - f_i$ fraction of the steps, link $i$ could have been successful but did not transmit in the original sequence. As the affectance is at most $a^{(t)}(i) \leq 1/2$, we conclude by easy calculus $\sum_j 1/d(s_j,r_i)^\alpha + \nu \leq 1/2\beta d(s_i,r_i)^\alpha$. From Lemma 1 we can then conclude that the success probability in these steps is at least $\exp(-1/2)$. For the remaining steps, we estimate the probability simply by $0$. Therefore, the reward gain is at least $(p_i - f_i) \cdot 2\exp(-1/2) - (1 - f_i) \leq \epsilon$.

This yields for all $i \in \mathrm{OPT}''$ and $\epsilon \leq 0.04$ that

$$\begin{aligned} p_i &\leq f_i + \frac{\epsilon + 1 - f_i}{2\exp(-1/2)} \\ &\leq \frac{1}{2}\left(1 + \frac{\exp(1/2)}{2}\right) + \frac{\epsilon \cdot \exp(1/2)}{2} \leq \frac{19}{20} \ , \end{aligned}$$

because $f_i \leq 1/2$. For $\hat{a}(i)$, we now get by definition of $q_i$

$$\hat{a}(i) \geq P_i \cdot 0 + (1 - P_i) \cdot \frac{1}{2} \geq \frac{1}{20} \cdot \frac{1}{2} = \frac{1}{40} \ .$$

Hence, we have

$$\hat{a}(i) = \sum_{j \in [n]} f_j a(j,i) \geq \frac{1}{40} \text{ for all } i \in \mathrm{OPT}''.$$

Taking the sum of all resulting inequalities, we get

$$\sum_{i \in \mathrm{OPT}''} \sum_{j \in [n]} f_j a(j,i) \geq \frac{|\mathrm{OPT}''|}{40}$$

or, equivalently,

$$\sum_{j \in [n]} f_j \left( \sum_{i \in \mathrm{OPT}''} a(j,i) \right) \geq \frac{|\mathrm{OPT}''|}{40} \ .$$

**Lemma 8** (Lemma 11 in [24])**.** *Assume $R$ is a feasible set under uniform power, such that for all $z \in R$, $\sum_{v \in R} a(z,v) \leq 2$. Then for any other link $u$, $\sum_{v \in R} a_u(v) = O(1)$.*

With Lemma 8 we have that $\sum_{i \in \mathrm{OPT}''} a(j,i) = O(1)$ for all $j \in [n]$ and hence

$$\sum_{j \in [n]} f_j = \Omega(|\mathrm{OPT}''|) \ .$$

□

Lemma 5 and 6 together yield that for any no-regret algorithm the expected number of successful transmissions converges to a constant fraction of the non-fading optimum. This directly proves Theorem 4. Together with the results from Section 5 we have proven an $O(\log^* n)$-approximation factor for no-regret learning for capacity maximization.

# 7 Simulation Results

In the sections before, we showed a close relation between the Rayleigh-fading and the non-fading models in theory. While bounds are given asymptotically for worst-case instances, our theoretical results can also be verified in simulations.

In particular, we consider Rayleigh and non-fading models with distance-based interferences and binary utilities based on a global feasibility threshold $\beta$ for the SINR. We examine the performance of an ALOHA-like protocol and the no-regret capacity-maximization algorithm. Simulations are carried out on random networks constructed by randomly placing receivers on a $1000 \times 1000$ plane. Each corresponding sender is placed by choosing the angle and the distance to the receiver uniformly at random from a fixed interval. This way, a minimal and a maximal distance between sender and receiver can be specified.

Comparing the Rayleigh-fading and the non-fading model the simulations show that the number of successful transmissions under uniform powers behave similarly when the sending probabilities are chosen uniformly, see Figure 1. The simulation was done (and the results averaged) over $40$ different networks with $100$ links each. For each network we considered $25$ different seeds for the randomizer to determine whether a link transmits. The SINR parameters were set to $\beta = 2.5$, $\alpha = 2.2$, and $\nu = 4 \cdot 10^{-7}$. The power for the uniform power assignment was set to $p_i = 2$ for all links $i$. For the square-root power assignment we set $p_i = 2\sqrt{d(s_i,r_i)^{2.2}}$. For the Rayleigh-fading channel we additionally used $10$ different seeds to determine whether a transmission is successful. The distance between a sender and the corresponding receiver was chosen between $20$ and $40$.

Figure 1 shows the number of successful transmissions averaged over all those runs. It assumes that the set of transmitting links is determined randomly with the same probability for each links to be transmitting. Neither the Rayleigh-fading model nor the non-fading model always predicts more success than the other one. The Rayleigh probability distribution leads to a smoothed curve compared to the non-fading model. This is due to the fact that even when the SINR constraint is not fulfilled in the non-fading model, the success probability in the Rayleigh-fading model still remains positive. On the other hand, when a transmission is definitely successful in the non-fading SINR model there is some probability for being not successful in the Rayleigh-fading model. The general characteristics of the curves are the same and show that the Rayleigh-fading and the non-fading model behave alike.

Choosing the optimal set of sending links under uniform powers, we reach on average $49.75$ successful transmissions in those networks.

The similarity can also be seen when taking a look at no-regret algorithms. Here we analyzed a version of the Randomized Weighted Majority Algorithm of Littlestone

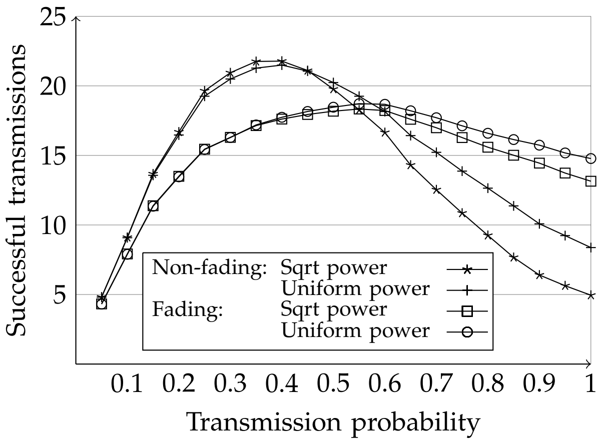


Fig. 1. Number of successful transmissions for different transmission probabilities under square-root and uniform power assignment and under the Rayleigh-fading and non-fading SINR model.

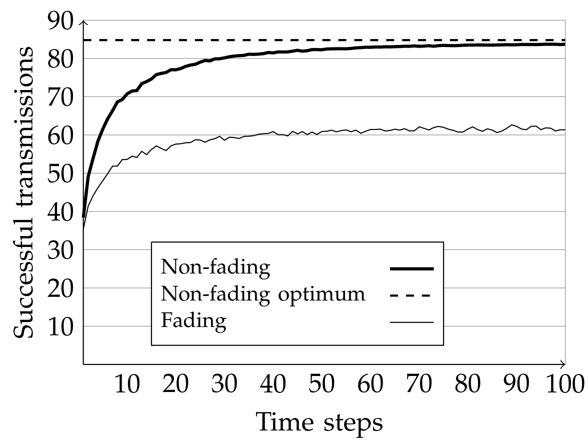


Fig. 2. Number of successful transmissions under the Rayleigh-fading and non-fading model when applying a no-regret algorithm.

and Warmuth [26]. The weights are initialized with 1 and multiplied by $(1-\eta)^{l_i}$ in every time step, where $l_i$ is the loss of not sending ($i=0$) or sending ($i=1$). The loss for sending and not being received is 1 and the loss of not sending at all is 0.5. In all other cases the loss is 0. These losses correspond to the utility function used in Section 6. The factor $\eta$ starts with $\sqrt{0.5}$ and is multiplied by $\sqrt{0.5}$ every time the number of time steps is increased above the next power of 2.

For the simulation shown in Figure 2 we used different networks with 200 links, distances between 0 and 100, $\beta = 0.5$, $\alpha = 2.1$, and $\nu = 0$. The other settings remained as before.

The results behave in the same way as observed by Ásgeirsson and Mitra [24] in their simulations. The Rayleigh-fading model shows more fluctuations due to its stochastic nature. We can also see that the no-regret algorithm converges quite quickly near the optimum of the non-fading model. The number of time steps needed for convergence depends on the specific instance, but a good performance can already be seen after 30 to 40 time steps.

## 8 Discussion and Open Problems

In this paper we showed that from an algorithmic point of view, the non-fading and the Rayleigh-fading model behave similarly in theory as well as in simulations. We regard this as a promising result because it indicates that existing results on approximation algorithms within non-fading models seem to apply more generally. Turning to a different, more realistic scenario does not create a fundamentally new situation as was the case when shifting from graph-based interference models to SINR-based ones.

Future research could take two different directions from this point. On the one hand, it could focus on the similarities, e.g., by improving the obtained bounds. Considering a particular situation, the $O(\log^* n)$-factor in Theorem 2 might be reduced to a constant, which we were not able to prove in general. Furthermore, the similarities could be exploited to take the best of the two worlds, in order to derive more sophisticated, hopefully distributed algorithms. On the other hand, also the differences could be taken into account. For example, the regret-learning simulation in the Rayleigh-fading model reaches a smaller capacity. It would be interesting to see if this is a general effect of the stochastic model or under which conditions this behavior can be observed.

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