Some computability exercises:

- Suppose L_1 and L_2 are decidable. Show that the complement of the intersection (i.e., $\overline{L_1 \cap L_2}$) is decidable.
- Use the Theorem of Rice to show that the following language is undecidable:

$$L = \{\langle P \rangle : \text{If } P \text{ halts, it outputs a word from } \{0, 00, 000, 0000, \ldots\}\}$$

- Show that the following language is undecidable (via Theorem of Rice or direct reduction):

$$L = \{\langle P \rangle : P \text{ accepts at least one word of odd length}\}$$

- Use the Theorem of Rice to show that the following language is undecidable:

$$L = \{\langle P \rangle : P \text{ halts for no input word with } |w| = 5\}$$

- Prove or disprove the decidability of the following language:

$$L = \{\langle P \rangle : P \text{ accepts all words of length at most } 3\}$$

- Suppose $L_3 = L_1 \cap L_2$ such that $L_3 \neq \emptyset$. L_1 is decidable, L_2 is not decidable. Is L_3 always decidable?
- Which of the following five languages L are undecidable?
  - $L = \Sigma^*$
  - $L = \{\, a_1 \ldots a_n \in \Sigma^* \mid n \in \mathbb{N} \text{ and } a_1 + \cdots + a_n = 9 \,\}$
  - $L = \{\, \langle P \rangle w \mid P \text{ halts with input } w \text{ after at most } |w|^2 \text{ steps} \,\}$
  - $L = \{\, \langle P \rangle w \mid P \text{ halts with input } w \text{ after at least } |w|^2 \text{ steps} \,\}$
  - $L = \{\, \langle P \rangle \mid P \text{ outputs 1 for each } w \text{ that represents some } n \in \mathbb{N} \text{ in binary encoding} \,\}$
- Which of the following statements are true for all languages L, L'?
  - If L is decidable, then $\overline{L}$ is recognizable.
  - If there is a reduction $L \leq L'$ and L' is decidable, then L is decidable.
  - If $L \subseteq L'$ and L' is decidable, then L is decidable as well.
  - If $L \subseteq L'$ and L is undecidable, then L' is undecidable as well.
- We are given a chain of reductions $L_1 \leq L_2 \leq \ldots \leq L_{k-1} \leq L_k$. Suppose $L_{k/2}$ is undecidable. What can you say about $L_{k/2+1}, \ldots, L_k$? What about $L_1, \ldots, L_{k/2-1}$?