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Übungen zur Vorlesung

Randomized Algorithms

Sommersemester 2007

Blatt 5

Exercise 1:
Consider the following language:

$$L = \{(G = (V, E), s \in V, t \in V) \mid s \text{ and } t \text{ are connected in } G\} \ .$$

a) First consider the undirected variant, i. e., G is undirected. Describe a probabilistic algorithm that uses only logarithmic space and expected polynomial time and, for an instance x outputs YES with probability at least $1/2$ if $x \in L$ and outputs NO with probability 1 otherwise. (Note that it takes logarithmic space to store a single node. The input itself is, of course, not included in this space bound.)

b) Now consider the directed variant, i. e., G is directed. Describe an algorithm with the same properties with the exception that the expected running time may be arbitrary (but bounded).

Exercise 2:
Show that adding edges may both increase and decrease the hitting time:

a) Construct two graphs $G = (V, E)$ and $G' = (V, E')$ with $E' \supseteq E$ such that for two nodes $v, w \in V$, $h'_{vw} = \Omega(n) \cdot h_{vw}$.

b) Construct two graphs $G = (V, E)$ and $G' = (V, E')$ with $E' \supseteq E$ such that for two nodes $v, w \in V$, $h'_{vw} = O(\log n/n^2) \cdot h_{vw}$.

Here, h_{vw} and h'_{vw} are the hitting times in G and G', respectively.

Exercise 3:
Construct a directed graph $G = (V, E)$ such that the cover time $C(G)$ is finite, but exponential in $|V|$.