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Übungen zur Vorlesung

Randomized Algorithms

Sommersemester 2007

Blatt 2

Exercise 1:

The ship of the drunken sailors sunk during a storm and an even number of n sailors stranded on a small island and got captured by cannibals. The cannibal chieftain is a fair one and gives each sailor a chance to free himself. Therefor he has to win the following game: Each sailor has to write down his (unique) name on a small note. Then each note is put i.u.r. in a small chest. Then each sailor is allowed to look in $\frac{n}{2}$ chests and has to tell the chieftain the number of a chest. The sailor wins only if the chest he chose contains the note with his name.

a) One sailor opens the first $\frac{n}{2}$ chests. Which is his chance to win this game?

b) Another sailor opens $\frac{n}{2}$ chests at random. Is this a better strategy?

c) A third sailor has another idea. He assigns randomly one chest to each sailor and starts opening the chest that he assigned to himself. The next chest he opens is the chest that is assigned to the sailor whose name he found in the opened chest and so on. What is to say about this idea?

Exercise 2:

The numbers $\mathrm{s}(k, r)$ are called Stirling numbers of the first kind and count all possible permutations of k numbers with exactly r cycles.

a) How many different permutations of k numbers with one cycle exist, equivalently what is $\mathrm{s}(k, 1)$?

b) Show that the probability that a random permutation of k numbers has a cycle of length more than $\frac{k}{2}$ is

$$\sum_{i>k/2}^{k} \frac{1}{i}. \tag{1}$$

c) Show that

$$\lim_{k\to\infty} \sum_{i>k/2}^{k} \frac{1}{i} = \ln 2. \tag{2}$$

Hint: Let H_n denote the n-th harmonic number. It is known that

$$\lim_{n\to\infty} (H_n - \ln n) = \gamma$$

where γ is the Euler-Mascheroni constant.[1]

[1] $\gamma \approx 0,5772156649015328606065120900824024310421593 35\ldots$

Exercise 3:

Assume the cannibals from Exercise 1 are very hungry and change the rules of the game as follows: All n sailors were eaten unless every sailor finds indepently the chest with his name inside. One of the sailors claims that there is a strategy that gives them a much higher probability to survive than the obvious $(1/2)^n$. Explain and analyze his idea.