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Übungen zur Vorlesung

Randomized Algorithms

Sommersemester 2007

Blatt 1

Exercise 1:
Suppose there are n totally drunken sailors returning to their ship and choosing their cabins independently, uniformly at random (i.u.r.)[1]. What is the expected number of sailors sleeping in their own cabin?

Exercise 2:
3-Coloring is a decicion problem, but we can phrase it as an optimization problem as follows. Suppose we are given a graph $G = (V, E)$, and we want to color each node with one of three colors, even when we aren't necessarily able to give different colors to every pair of adjacent nodes. Rather we say that an edge (u, v) is *satisfied* if the colors assigned to u and v are different.
Consider a 3-coloring that maximizes the number of satisfied edges, and let c^* denote this number. Give a polynomial-time algorithm that produces a 3-coloring that satisfies at least $\frac{2}{3}c^*$ edges. If you want your algorithm can be randomized; in this case the *expected* number of edges it satisfies should be at least $\frac{2}{3}c^*$.

Exercise 3:
Suppose you are given a coin for which the probability of heads, say p, is unknown. How can you use this coin to generate unbiased (i.e., $\Pr[\text{heads}] = \Pr[\text{tails}] = \frac{1}{2}$) coin-flips? Give a scheme for which the expected number of flips of the biased coin for extracting one unbiased coin-flip is no more than $\frac{1}{p(1-p)}$.

Exercise 4:
In a quiz show three participants can win a trip to Hawaii if they win the game with rules as follows. Each participant gets i.u.r. either a red or a green hat, he can't see the color of his own but the colors of the two others. Then all three write down either "red", "green" or "unknown". The three players win if at least one player wrote down "red" or "green" and all players that wrote down "red" or "green" correctly guessed the color of their hat.

a) Give a strategy for the three players that guarantees a chance for wining of exactly 1/2.

b) Is there a scheme that guarantees more than 50% winning probability? Defend your choice.

[1] Even drunk, they preserve their intimacy.